QUESTIONS FOR SOLUTION.

2572. (Proposed by Professor SYLVESTER.)—1. If $h, k, l \ldots$ are the distinct prime factors of n, prove that the number of points (which I term pluperfect points of the nth order) at which a cubic curve can have the highest degree of contact with a curve of the degree n (not composed of repetitions of a curve of lower degree) is $9n^2(1-h^{-2})(1-k^{-2})(1-l^{-2})$.

2. Show that if the first tree in the solution to Quest. 2473 (*Reprint*, Vol. VIII., p. 106) be planted at a pluperfect point of the nth order in a cubic, the sequence of tree-marks 1, 2. 4, 5, 7... may be replaced by recurring periods of $2n$ numerals, and that the two halves of each period will consist of the same n numerals arranged in reverse order, that in fact only the first n of the numerals 1, 2, 4.... need appear in the result.

3. Hence prove that n trees may be so arranged as to contain between them $E\left\{\frac{1}{6}n(n-1)\right\} - E\left\{\frac{1}{3}n\right\}$ rows of three in a row, where E (the symbol of entirety) denotes that the integer part only is to be taken of the function which it governs.

NOTE.—Thus, for 81 trees the number will be 1053 instead of 800, the number obtained when the first tree is at a non-pluperfect point; and for 15 trees the number is 30 instead of 26, the number stated in the Editorial reference at the end of the solution above cited as applicable to 15 points, but which is, in fact, the number for 14, according to the formula stated above. It must, however, be observed, that this formula, does not in general give the absolute maximum. Thus, for $n=9$ the formula gives only 9 rows, whereas the true maximum (dealing with *real* trees) is 10. In fact, $E\left\{\frac{1}{6}n(n-1)\right\}$ is the arithmetical limit obtained by dividing the number of duads of n by 3, the number of them in each row. Thus, $E\left\{\frac{1}{3}n\right\}$ is the difference between this limit and the actual number of the formula, which may also be expressed as equal to $\frac{1}{6}(n^2-3n)$ when this is an integer, and the integer immediately superior to it when it is fractional. Thus we see in a moment that with 1000 trees, the number of rows containing three in a row may be made equal to 166167. In a word, when the first tree is planted at an ordinary point in the cubic, the number of rows given by the formulæ in Quest. 2473 is $E\left\{\frac{1}{8}(n-1)^2\right\}$, when at a pluperfect point the number is

$$E\left\{\frac{1}{6}(n-1)(n-2)\right\}.$$

Another method of placing points on a cubic which gives 10 rows for 9 points, and possibly in all cases the true maximum, will form the subject of a future question.