

Poincare's Legacies, Part II

pages from year two of a mathematical blog

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one can view this coordinate system as determining a pair of Euler angles θ, λ (or a latitude and longitude) to be assigned to every point on one's original sphere.

(9) The above examples were all geometric in nature, but one can also consider "combinatorial" coordinate systems, which allow one to identify combinatorial objects with numerical ones. An extremely familiar example of this is enumeration: one can identify a set A of (say) five elements with the numbers 1,2,3,4,5 simply by choosing an enumeration a_1, a_2, \dots, a_5 of the set A . One can similarly enumerate other combinatorial objects (e.g. graphs, relations, trees, partial orders, etc), and indeed this is done all the time in combinatorics. Similarly for algebraic objects, such as roots of a subgroup H (or more generally, inverses of a group G); one can identify such a root with H itself by designating an element of that root to be the "identity" or "origin".

More generally, a coordinate system Φ can be viewed as an isomorphism $\Phi: A \rightarrow G$ between a given geometric (or combinatorial) object A in some class (e.g. a circle), and a standard object G in that class (e.g. the standard unit circle).

Coordinate systems identify geometric or combinatorial objects with numerical (or standard) ones, but in many cases, there is no natural (or canonical) choice of this identification; instead, one may be faced with a variety of choices, each of which is equally valid. One can of course just case there is no natural choice of objects, or one system to the others, then it becomes a matter of convenience to have a system (identifying the objects).

1.6. The Black-Scholes equation

- (1) A call option for S at time t_1 and at price P is a contract that gives the buyer of the option the right (but not the obligation) to buy a unit of S from the seller of P at time t_1 (conversely, the seller of the option has the obligation but not the right to sell a unit of the option at time t_1 , if the buyer so requests).
- (2) A put option for S at time t_1 and at price P is a contract that gives the buyer of the option the right (but not the obligation) to sell a unit of S to the seller of P at time t_1 (and conversely, the seller of the option has the obligation but not the right to buy a unit of S from the buyer of the option at time t_1 , if the buyer so requests).
- (3) More complicated options, such as straddles, can be formed by taking linear combinations of call and put options. One can also consider options which offer rights and obligations for rather than the "European options" described above, which only apply at a fixed time t_1 . The Black-Scholes theory has been applied to American options, but this is beyond the scope of this post.

The problem is this: what is the "correct" price to assign to any European option (such as a put or a call) at expiration time t_1 ? Of course, due to the volatility of the instrument S , the future price S_{t_1} of this instrument at time t_1 is not known at time t_0 . Nevertheless, and this is really what is at issue, it is still possible to compute deterministically the price of an option that depends on that unknown price, assuming one of which is that one knows the underlying instrument S .

1.6.1. How to compute price. Before we do this, we must first settle a fundamental financial question: what is the price of some asset A ? In most cases, the price would depend on many factors, such as the demand for A , transaction costs in buying or selling